Effect of frequency offset on orthogonality of loosely synchronous codes

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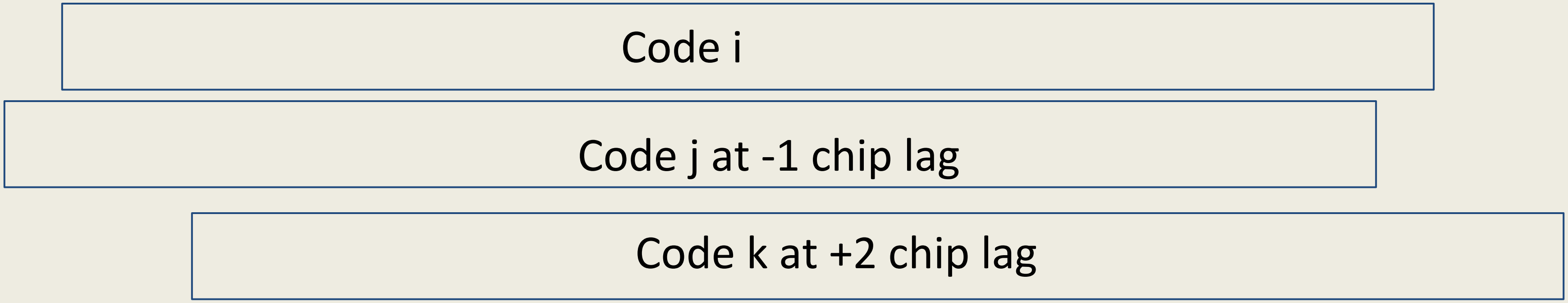
Motivation

- Reliable, ad hoc, low-power, multi-user communication in cluttered environments among near-ground agents
  - **Synchronization challenges**: intermittent GPS access, frequency offset, no power control; **tight time/frequency synchronization particularly difficult to achieve with software-defined radios**
  - **Solution**: Loosely synchronous (LS) codes; enable minimal multiple access interference (MAI) even in weakly sychronized regimes and/or power mismatch
- Exploit various frequency bands (e.g., low VHF, UHF) as part of a multi-wavelength hybrid system for robust low power communications in Army-relevant scenarios
  - Near-ground low frequency channels provide superior penetration,reduced multipath, and much smaller frequency offsets than at microwave
  - Recent advance in miniature antennas enable practicality

Background on LS codes

- Typical DS-CDMA codes have non-zero (auto/cross) correlation at nonzero lag
  - Gold, Kasami, Walsh
- Challenges: ISI/MAI limited and near/far problem
  - Conventional solution: power-control, interference cancellation, multi-user detectors (MUD);
    - Infrastructure-dependent; costly/power hungry
- Exist codes with a zero correlation zone (ZCZ)
  - off-peak aperiodic correlation = 0
  - Within ZCZ: zero ISI and zero MAI ⇒ single-user-like communications performance *without* MUD

Codes with ZCZ



Code i

Code j at -1 chip lag

Code k at +2 chip lag

- ZCZ: A set of lags {-L, -L+1, …, L-1, L} for which the correlation is exactly zero. For autocorrelation, $0 \notin$ ZCZ.
- Define: C = a set of codes having a ZCZ. |L|=max value in ZCZ. M = code length. |C|= size of code family.
- A bound from [1] establishes that for any C:
  - $|C| * (|L|+1) \leq M$
  - I.e., for fixed M, number of codes with ZCZ is limited

[1] P. Z. Fan, "Spreading sequence design and theoretical limits for quasisynchronous CDMA systems," EURASIP J. Wireless Comm. and Networking, vol. 2004, no. 1, pp. 19–31, 2004.

Codes with ZCZ

- For ad hoc networks, assume only intermittent time synchrony
  - E.g., nodes synchronize approximately every 10 s
  - Exists on order of 10 µs time uncertainty among nodes
    - Clock drift (1 µs / s) + processing delay of sync signal
- An example: BW= 1.25 MHz, 10 µs ⇒ |L|=13 for a single-carrier system
  - For |C|=16, code length M > 200
  - Number of codes |C|=4, for M=64

Codes with ZCZ

Challenges:

- Problem 1: Large |L| forces one to
  - increase code length M (for |C|=16, M > 200)
  - decrease number of codes |C| (for M=64, |C| = 4)

- Problem 2: Extending the codes in time ⇒ more susceptible to orthogonality loss due to frequency offset

$$\sum x(n)\cdot y(n+l)=0 \xrightarrow{\text{Freq offset}} \sum x(n)\cdot y(n+l)e^{j\cdot \Delta w\cdot (n+l)} \neq 0$$

Approach and analysis:

- Use multiple carriers
  - E.g., with BW ≅1.25 MHz, 8 subcarriers, one can use a ZCZ with |L|=3 chips which covers ≅ 20 µs
  - |L|=3 ⇒ LS code family with length 67 codes can support 16 users

- The effect of frequency offset is investigated for various families of codes

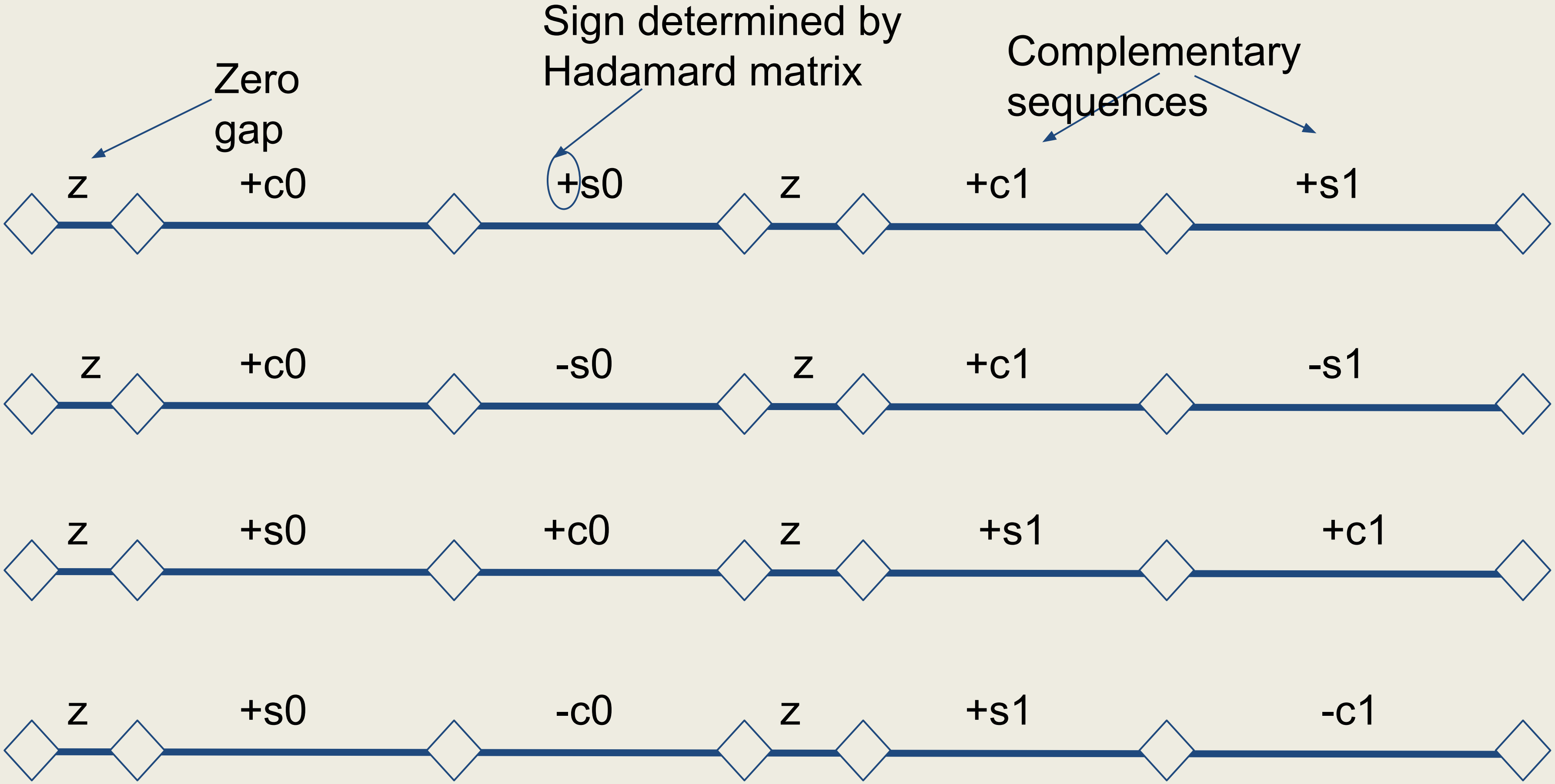
Construction of LS Codes

General construction technique depends on choice of

1. Complementary sequences / mates
   - → Form code building blocks
   - → Ensure orthogonality at non-zero lags within ZCZ
2. Hadamard matrix
   - → Ensures orthogonality at zero lag
3. Zero gaps
   a. Prevent intersymbol interference
   b. Prevent overlap between mates

[2] S. Stanczak et Al. . Are LAS-codes a miracle?. in Global Telecommunications Conference, 2001. GLOBECOM'01. IEEE (Vol. 1, pp. 589-593). IEEE.

ZCZ code example



Zero gap

Sign determined by Hadamard matrix

Complementary sequences

z +c0 +s0 z +c1 +s1

z +c0 -s0 z +c1 -s1

z +s0 +c0 z +s1 +c1

z +s0 -c0 z +s1 -c1

Example of 4 code ZCZ family with ZCZ duration=z

Tradeoffs

- The above procedure is parameterized by:
  - Choice of complementary sequence (CS)
  - Method of interleaving complementary sequences
  - Choice of Hadamard matrix
- Each choice results in different instances of a ZCZ code family
- Do these instances differ in terms of the frequency offset induced orthogonality loss?

Notations

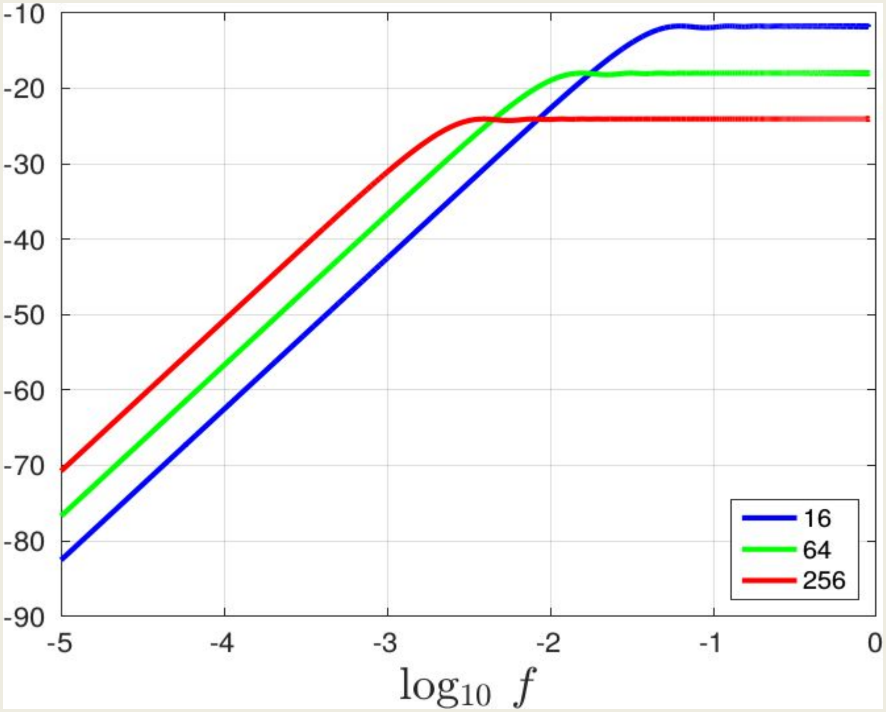
$$\mathbf{R}_{k,l} = \sum_{n=0}^{L-1} \mathbf{c}_k(n)\ \mathbf{c}_l^*(n)e^{-j\omega n}$$

- The cross-correlation between codes c_k and c_l in the presence of frequency offset w. Codes are of length L.

$$\rho(\mathcal{S},\omega,q) \triangleq \left(\frac{1}{2\binom{K}{2}}\sum_{k}\sum_{l,l\neq k}|\mathbf{R}_{k,l}|^q\right)^{\frac{1}{q}}$$

- The “q-norm” cross-correlation, averaged across all codes in set S, at frequency offset w. Indicies k and l range over all users in the system and K is the total number of users.

Orthogonality loss due to frequency offset (Hadamard)



Theorem 1: Suppose that

1. Code elements are of constant modulus
2. Number of users equals code length
3. On average, all active users have same transmit power

→ Then, $\rho(S,w,2)$ ("2-norm") is *independent* of code choice S

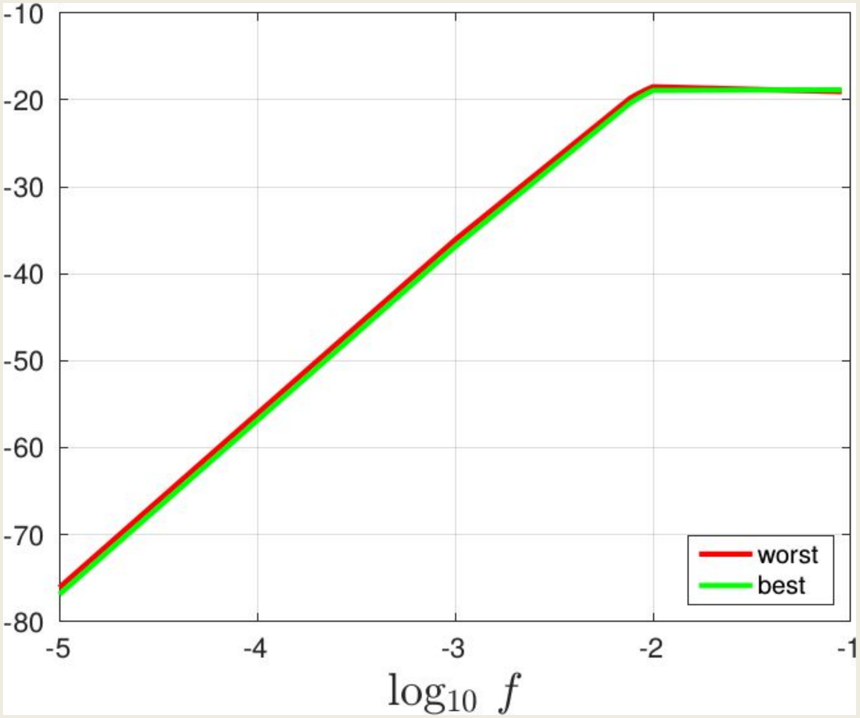
Average MUI for Hadamard codes of various lengths shown in legend. X-axis: normalized frequency offset (frequency times chip time). Y axis: 10 $\log_{10}(\rho(S,w=2\pi f,2)^2 / L^2)$, which is the average MUI (dB) presented by a single interferer on a link of interest. In absence of frequency offset, MUI (dB) is -∞.

ZCZ Code Simulation Study

- Randomly generate 1000 Hadamard matrices H
- For each H and each valid CS interleaving scheme C
  - Construct the ZCZ code family using H and C
    - 16 total codes
    - each of length 64+3
  - For each fixed frequency offset
    - Find all pair-wise cross correlations at all shifts within the ZCZ
    - Compute $\rho(S,w,1)$, $\rho(S,w,2)$, and $\rho(S,w,\infty)$

For each norm, return the best and worst found families

Orthogonality loss due to frequency offset (ZCZ codes)

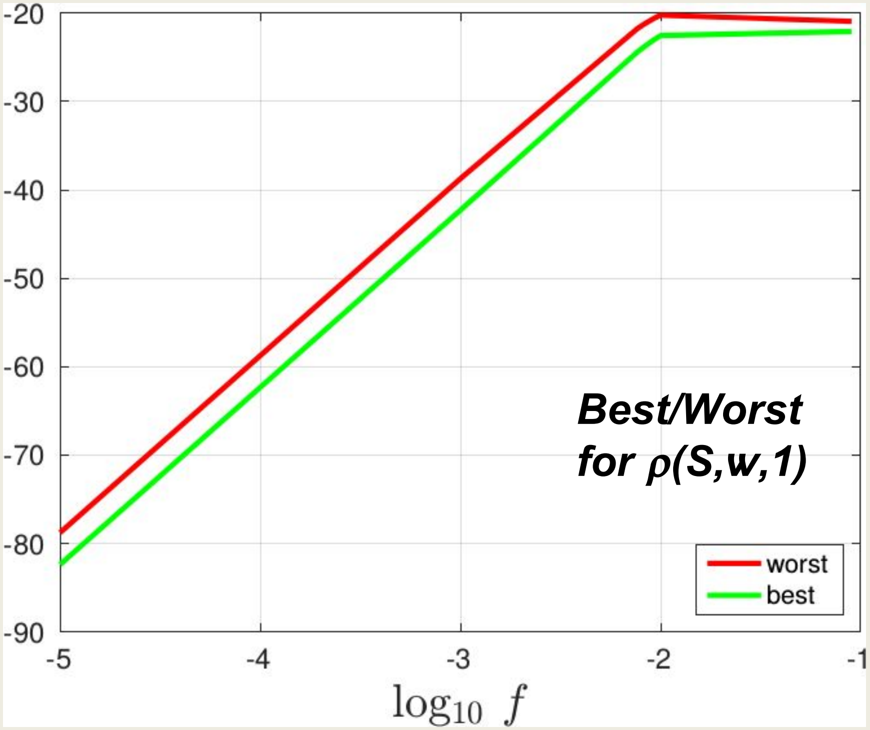


- ZCZ codes are not constant modulus, so Theorem 1 does not apply directly

- However, for small z, most elements are non-zero, so we expect it to approximately hold

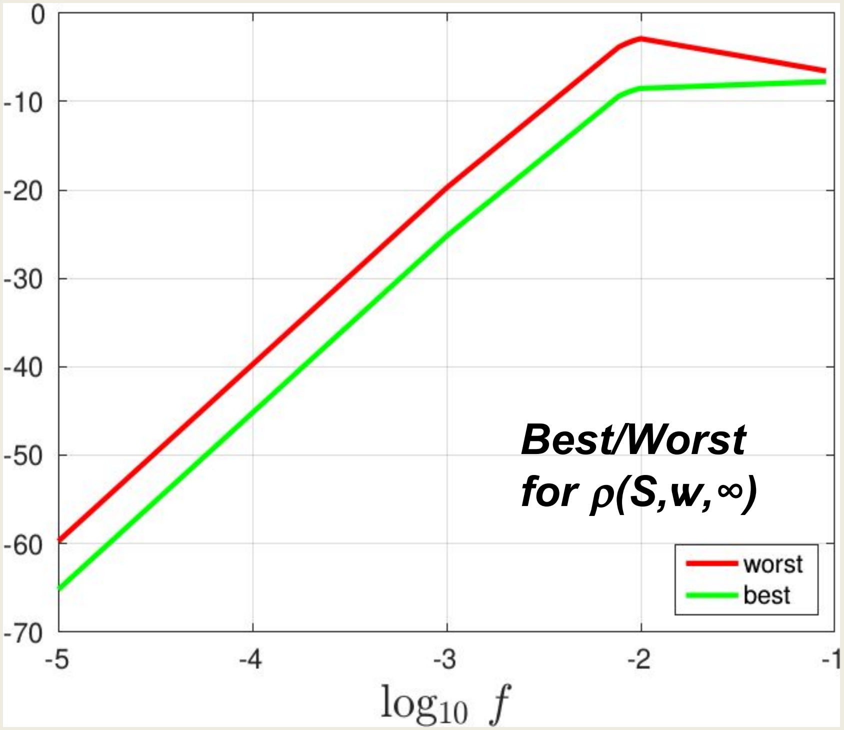
Average MUI for best and worst LS codes of length 64+3.. X-axis: normalized frequency offset (frequency times chip time). Y axis: 20 log10($\rho(S,w=2\pi f,2)$ / L) presented by a single interferer on a link of interest, normalized by code energy. Best and worst curves separated by 0.8 dB.

Orthogonality loss due to frequency offset (ZCZ codes)



Y axis: 20 $\log_{10}$($\rho(S,w=2\pi f,1)$ / L)

$\rho(S,w,1)$: at $\log_{10}$(f)=-3, difference of ~3.7 dB $\rho(S,w,\infty)$: at $\log_{10}$(f)=-3, difference of ~1.1 dB



Y axis: 20 $\log_{10}$($\rho(S,w=2\pi f,\infty)$ / L)

$\rho(S,w,\infty)$: at $\log_{10}$(f)=-3, difference of ~5.6 dB $\rho(S,w,1)$: at $\log_{10}$(f)=-3, difference of ~1.2 dB

Conclusions

- L2 norm: all discovered families are nearly identical
  - useful for Gaussian interference process, i.e., when there are a large numbers of users
  - In this case, CLT can be invoked, and L2 norm captures variance
- For small numbers of users (e.g. in LS codes), other norms may be more relevant
  - CLT cannot be invoked
  - Power of interferers may follow other laws, e.g. exponential
- L1 norm: nearly 4 dB difference
- L-∞ norm: nearly 6 dB difference
  - captures worst-case interference a single interferer can provide

Conclusions (Cont’d)

- We considered only real-valued complementary sequences, real-valued Hadamard matrices, and fully loaded systems (number of users = code length)
  - For LS codes, using complex complementary sequences/Hadamard matrices may offer advantages
- For Hadamard codes, if number of users < code length, then Theorem 1 does not hold
  - Good news: some codes are better than others
  - Essentially, one trades off a smaller number of users for codes with better performance with respect to $\rho(S,w,2)$

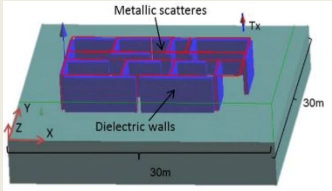
BACKUP

Motivation

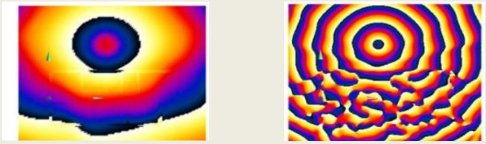
- Tactical mobile ad hoc network performance limited by adverse propagation & lack of infrastructure
- Exploit various frequency bands (e.g., low VHF, UHF) as part of a multi-wavelength hybrid system for robust low power communications in Army relevant scenarios
  - Near-ground low frequency channels provide superior penetration and reduced multipath
  - Recent advance in miniature antennas and channels studies

Potential for persistent low power, low complexity Multi-user communications and Networking in austere infrastructure poor environments

Simulation Example



Phase variation at z = 1m



20MHz

100 MHz

Conventional vs. Miniature antennas